

3-Consider the mass–spring–damper system of Figure 1, which may be subject to two inforces  $u_1(t)$  and  $u_2(t)$ . Show that the displacements  $x_1(t)$  and  $x_2(t)$  of the two masses are given by

$$K_1 = 1, k_2 = 4, m_2 = 4, B = 2$$

$$u_1(t) = \sin t, u_2(t) = u \sin t$$

$$x_1 = x_2 = \dot{x}_1 = 0, x_2 = 2, t = 0$$

$$M_1 \frac{d^2 x_1}{dt^2} + k_1(x_1) + B_1(\dot{x}_1 - \dot{x}_2) = u_1(t)$$

$$M_1 \frac{d^2 x_1}{dt^2} = -k_1(x_1) + B_1(\dot{x}_2 - \dot{x}_1) + u_1(t)$$

$$M_2 \frac{d^2 x_2}{dt^2} = +k_2 x_2 + B_1(\dot{x}_2 - \dot{x}_1) + u_2(t)$$

$$M_2 \frac{d^2 x_2}{dt^2} = -k_1 x_1 - B_1(\dot{x}_2 - \dot{x}_1) + u_2(t)$$

$$\frac{d^2 x_1}{dt^2} = -k_1 x_1 + 2(\dot{x}_2 - \dot{x}_1) + \sin t$$

$$u \frac{d^2 x_1}{dt^2} = -u x_2 - 2(\dot{x}_2 - \dot{x}_1) + u \sin t$$

$$\frac{d^2 x_1}{dt^2} = x_1 t + 2(x_2 - x_1) - \cos t + c$$

$$x_2 = \frac{dx_2}{dt} \int \frac{dx_2}{dt} = x_2$$

$$x_1 = -\frac{x_1 t^2}{2} + 2(x_2 - x_1) - \sin t + c_1 + c_2$$

$$t = 0, \quad x_1 = x_2 = 0$$

$$c_2 = 0,$$

$$t = 0, \quad x_1 = 0, x_2 = 2, x_1 = x_2 = 0$$

$$0 = 0 + 2(0 - 0) - 1 + c_1 c_1 = 1$$

$$x_1(t) = -x_1 \frac{(t)^2}{2} + 2x_2(t)t - 2x_1(t)t - \sin t + 1(t)$$

$$x_1(t) = \frac{t^2}{2} + 2(x_1(t)t + x(t) + 2x_1(t)t - \sin t + (t)$$

$$x_1(t) = (\frac{t^2}{2} + 2t + 1)2x_2(t)t - \sin t + (t)$$

$$u \frac{d^2 x_2}{dt^2} = \Sigma - ux_2 t - 2(x_2 - x_1) - u \cos + c_1$$

$$u_1 x_2 = -u \frac{d^2 x_2}{dt^2} = \Sigma - ux_2 t - 2(x_2 - x_1) - u \cos + c_1$$

$$u_1 x_2 = -ux_2 \frac{t^2}{2} = -2(x_2 - x_1)t - u \sin + c_1 t + c_2$$

$$\text{then } t = 0, x_1 = x_2 = 0$$

$$\Rightarrow c_2 = 0$$

$$t = 0 \quad x_1 = x_2 = 0, x_1 = 0, x_2 = 2$$

$$u(x) = 0 - 0 - 4 - c_1$$

$$c_1 = 8 + 4$$

$$c_1 = 12$$

$$4(x_2(t) = -2(x_2(t))t^2 - 2(x_2(t)t + 2x(t) - 4 \sin t + 12t$$

$$2x_2(t)t^2 + 2x_2(t)t + (x_2(t)) = 2x_1(t) \quad t - 4 \sin + 12t$$

$$\frac{2x_1(t) \quad t - 4 \sin + 12t}{2(t^2 4t + 2)} : -$$

$$x_1(t)\left(\frac{t^2}{2} + 2t + 1\right) = \left(\frac{2x_1(t) - t - 4\sin t + 12t}{(t^2 + t + 2)}\right) - \sin t + t$$

$$x_1(t)\left(\frac{t^2}{2} + 2t + 1\right) = \frac{2x_1(t) - t - 4\sin t + 12t}{(t^2 + t + 2)} - \sin t + t$$

$$x_1(t)\left(\frac{t^2}{2} + 2t + 1\right) = -\frac{2x_1(t) - t - 4\sin t + 12t}{(t^2 + 2t + 2)} - \sin t + t$$

$$x_1(t)\left(\frac{t^2}{2} + 2t + 1\right) = -\frac{2x_1(t) - t}{(t^2 + 2t + 2)} = \frac{-u\sin t + 12t - (\sin t - t)(t^2 + t + 2)}{(t^2 + 2t + 2)}$$

$$x_1(t) = \frac{-u\sin t + 12t - (\sin t - t)(t^2 + t + 2)}{((\frac{t^2}{2} + 2t + 1)((t^2 + t + 2) - 2t)} \implies x_1(t) = 2\frac{-u\sin t + 12t - (\sin t - t)(t^2 + t + 2)}{((\frac{t^2}{2} + 2t + 1)(t^2 + t + 2) - 2t)}t - u\sin t + 12t =$$